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Analogy between fluid friction and heat transfer in annuli†

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Abstract—A semi-theoretical equation for the heat transfer coefficient on the inner wall in turbulent annular flow is obtained from the analogy between fluid friction and heat transfer. Comparison of the equation with experimental data of air flow obtained by the author shows good agreement. It is also in close agreement with the equation proposed by Monrad and Pelton at a higher Prandtl number.

INTRODUCTION

THERE are many experimental or recommended equations for heat transfer to annuli. But the values of heat transfer coefficients given by these equations differ considerably, and all of them are higher than the author's experimental values for air.

The author extended the concept of the analogy between heat transfer and fluid friction to apply it to this case and obtained a semi-theoretical equation, which could correlate the data for air, and was also in close agreement with Monrad and Pelton's equation [1] if applied for liquid.

VELOCITY DISTRIBUTION AND FLUID FRICTION

The velocity distribution and fluid friction in annuli were well discussed by Rothfus *et al.* [2]. They concluded that the radius of maximum velocity in fully developed turbulent flow was the same as predicted by Lamb's equation [3] for entirely viscous flow

$$r_m = \{(r_1^2 - r_0^2)/2\ln(r_1/r_0)\}^{1/2}. \tag{1}$$

Furthermore, according to them

$$f_1 = \frac{2\tau_1}{\rho \bar{V}_1^2} = 0.0791\left\{\frac{2(r_1^2 - r_m^2)\bar{V}_1\rho}{r_1\mu}\right\}^{-1/4} \tag{2}$$

$\bar{V}_1$ seems to be more reasonable than $\bar{V}$.

Between the skin frictions on outer and inner tube walls there exists the following relation:

$$\tau_0 r_0/(r_m^2 - r_0^2) = \tau_1 r_1/(r_1^2 - r_m^2). \tag{3}$$

When f_0 is defined as based on the overall average velocity $\bar{V}$, and Re as $d_{eq}\bar{V}\rho/\mu$

$$f_0 = K_1\,Re^{-1/4}$$

$$K_1 = 0.0791\frac{r_1}{r_0}\frac{r_m^2 - r_0^2}{r_1^2 - r_m^2}\left\{\frac{r_1^2 - r_m^2}{r_1(r_1 - r_0)}\frac{\bar{V}_1}{\bar{V}}\right\}^{-1/4}\left(\frac{\bar{V}_1}{\bar{V}}\right)^2. \tag{4}$$

Furthermore, Rothfus *et al.* discussed the velocity distribution in the annuli in detail. The author, however, assumes Kármán–Prandtl's seventh root law for brevity, and the ratios of V_m, $\bar{V}_1$, and $\bar{V}_0$ to $\bar{V}$ can then be calculated as follows:

$$\frac{V_m}{\bar{V}} = \frac{60}{7}\frac{r_1 + r_0}{7r_1 + 7r_0 + r_m} \tag{5}$$

$$\frac{\bar{V}_1}{\bar{V}} = \frac{(r_1 + r_0)(7r_1 + 8r_m)}{(r_1 + r_m)(7r_1 + 7r_0 + r_m)} \tag{6}$$

$$\frac{\bar{V}_0}{\bar{V}} = \frac{(r_1 + r_0)(7r_0 + 8r_m)}{(r_0 + r_m)(7r_1 + 7r_0 + r_m)}. \tag{7}$$

HEAT TRANSFER COEFFICIENTS TO ANNULI

Firstly, the following case is considered when the radii of maximum temperature and maximum velocity are equal. The heat is transferred from the region inside r_m to the inner tube wall, and from outside r_m to the outer tube, the transfers being independent of each other. Then, the relation between skin friction on the inner tube wall and heat transferred to the inner tube wall is

$$(t_b - t_0)/q_0 = (\mu/k)(V_b/\tau_0). \tag{8}$$

In the turbulent core, the Reynolds analogy is applied and the following relation is obtained:

$$(\bar{t}_0 - t_b)/q_0 = (\bar{V}_0 - V_b)/(c_p\tau_0). \tag{9}$$

Secondly, the following case is to be considered. The outer tube wall is insulated, and the heat is transferred to the inner tube wall from all parts of the annulus. The resistance of the laminar layer to heat transfer is exactly the same as in the former case, therefore a similar equation to equation (8) is obtained

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NOMENCLATURE

c_p specific heat of fluid at constant pressure
d_{eq} equivalent diameter, $2(r_1 - r_0)$
f Fanning-type friction factor
h heat transfer coefficient
k thermal conductivity of fluid
K_1, K_2, K_3 constant depending on r_0/r_1 only
Nu Nusselt number, hd_{eq}/k
Pr Prandtl number
q heat transferred to wall
Re Reynolds number, $d_{eq}\bar{V}\rho/\mu$
r radial distance from the center to a point in the fluid stream
t local fluid temperature
$\bar{t}$ average fluid temperature
V local fluid velocity
$\bar{V}$ average fluid velocity.

Greek symbols
α eddy diffusivity of heat
$\Delta_a t$ temperature change along the axial direction
μ absolute fluid viscosity
ρ fluid density
τ skin friction.

Subscripts
0 inner tube wall or region inside radius of maximum velocity
1 outer tube wall or region outside radius of maximum velocity
b boundary between laminar layer and turbulent core
m maximum velocity.

$$(t_b - t_0)/q = (\mu/k)(V_b/\tau_0). \tag{10}$$

The resistance in the turbulent core, however, is not the same. On the following assumptions the ratio of the resistances of the two cases can be calculated: that the velocity in the turbulent core has the uniform value $\bar{V}$; that the temperature change along the axial direction, $\Delta_a t$, is equal over the entire annulus; and that in the turbulent core, the eddy diffusivity of heat α is constant across the annulus. In the first case, heat passing through the section r is expressed as

$$\alpha(\mathrm{d}t/\mathrm{d}r)2\pi r = \pi(r_m^2 - r^2)\bar{V}\rho c_p \Delta_a t. \tag{11}$$

When this is solved the average temperature difference can be calculated. On the other hand, q_0 is expressed as $\pi(r_m^2 - r_b^2)\bar{V}\rho c_p \Delta_a t$ and hence, the thermal resistance in the turbulent core is

$$\frac{\bar{t}_0 - t_b}{q_0} = \frac{1}{\pi(r_m^2 - r_b^2)^2 \alpha} \times \left\{\frac{r_m^2 r_b^2}{2} - \frac{3r_m^4}{8} - \frac{r_b^4}{8} + \frac{r_m^4}{2}\ln\left(\frac{r_m}{r_b}\right)\right\}. \tag{12}$$

In the second case, a similar result is obtained and therefore the ratio of the resistances of the two cases is

$$N = \{(\bar{t} - t_b)/q\}/\{(\bar{t}_0 - t_b)/q_0\}$$

$$= \left(\frac{r_m^2 - r_b^2}{r_1^2 - r_b^2}\right)^2 \left\{\frac{r_1^2 r_b^2 - \dfrac{3r_1^4}{4} - \dfrac{r_b^4}{4} + r_1^4 \ln\left(\dfrac{r_1}{r_b}\right)}{r_m^2 r_b^2 - \dfrac{3r_m^4}{4} - \dfrac{r_b^4}{4} + r_m^4 \ln\left(\dfrac{r_m}{r_b}\right)}\right\}. \tag{13}$$

Consequently, the thermal resistance of the turbulent core in the second case becomes

$$(\bar{t} - t_b)/q = N(\bar{V}_0 - V_b)/(c_p \tau_0). \tag{14}$$

Combining equations (10) and (14)

$$h = \frac{(f_0/2)c_p \rho \bar{V}}{N(\bar{V}_0/\bar{V} - V_b/\bar{V}) + Pr(V_b/\bar{V})}. \tag{15}$$

According to Prandtl [4], and by application of the same correction as introduced by the author [5] into Prandtl's equation of the analogy, the following relations are obtained:

$$V_b/\bar{V} = K_2\sqrt{f_0}/Pr^{0.2}$$

$$K_2 = \frac{V_m}{\bar{V}}\left(\frac{1}{K_1^4}\frac{4}{7}\frac{r_1 - r_0}{r_m - r_0}\frac{V_m}{\bar{V}}\right)^{1/6}. \tag{16}$$

Furthermore, r_b/r_1 can be calculated from the following equation:

$$(r_b - r_0)/(r_1 - r_0) = K_3 f_0^{7/2}/Pr^{0.2}$$

$$K_3 = 4K_2/K_1^4 \tag{17}$$

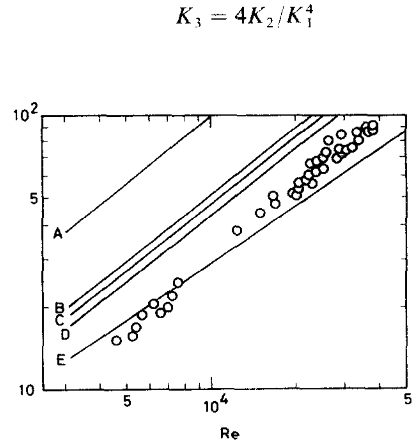

FIG. 1. Comparison of several equations for heat transfer to annuli, $Pr = 0.74$, $r_0/r_1 = 0.457$: A, Foust and Christian; B, Davis; C, Wiegand; D, Monrad and Pelton; E, equation (15).

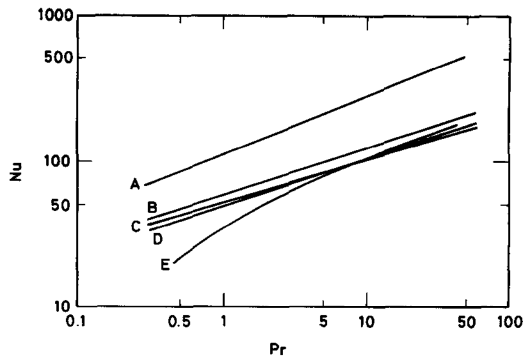

FIG. 2. Comparison of several equations for heat transfer to annuli, $Re = 10\,000$, $r_0/r_1 = 0.457$: A, Foust and Christian; B, Davis; C, Wiegand; D, Monrad and Pelton; E, equation (15).

and consequently, the heat transfer coefficient h can be calculated from equation (15).

In Fig. 1, the plots are the author's data for air flow ($Pr = 0.74$) in an annulus ($r_0/r_1 = 0.457$) for various Reynolds numbers. Equation (15) correlates the data fairly well and for comparison, several equations (Monrad and Pelton [1], Foust and Christian [6], Davis [7], and Wiegand [8]) are shown, all of which give too high values.

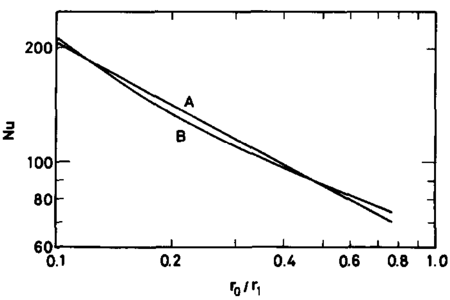

FIG. 3. Comparison of equations, $Re = 10\,000$, $Pr = 7$: A, Monrad and Pelton; B, equation (15).

In Fig. 2, equation (15) and Monrad and Pelton's equation

$$Nu = 0.020(r_1/r_0)^{0.53}\,Re^{0.8}\,Pr^{1/3} \qquad (18)$$

are compared at the same radius ratio $r_0/r_1 = 0.457$, and Reynolds number $Re = 10\,000$, only the Prandtl number being a variable; they are in close agreement for $Pr > 5$. It should be noted that the effect of N becomes smaller with an increase of Pr. The satisfactory agreement of the two equations can be seen by comparing them when $Pr = 7$ and $Re = 10\,000$, and r_0/r_1 is a variable as in Fig. 3.

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ANALOGIE ENTRE FROTTEMENT DU FLUIDE ET TRANSFERT DE CHALEUR DANS LES ESPACES ANNULAIRES

Résumé—Une équation semi-théorique pour le coefficient de transfert thermique à la paroi interne est obtenue pour un écoulement turbulent, à partir de l'analogie entre frottement et transfert de chaleur. La comparaison entre le calcul et les données expérimentales pour un écoulement d'air, obtenues par l'auteur, montre un bon accord. Il y a aussi une concordance avec l'équation proposée par Monrad et Pelton pour les nombres de Prandtl élevés.

ANALOGIE ZWISCHEN REIBUNG UND WÄRMEÜBERGANG BEI DER STRÖMUNG EINES FLUIDES IN EINEM RINGSPALT

Zusammenfassung—Aus der Analogie zwischen Reibung und Wärmeübergang läßt sich eine halb-theoretische Gleichung ableiten, mit der der Wärmeübergangskoeffizient an der inneren Wand eines turbulent durchströmten Ringspalts berechnet werden kann. Ein Vergleich der berechneten mit eigenen gemessenen Werten für Luft zeigt eine gute Übereinstimmung. Die ermittelte Gleichung zeigt ebenfalls gute Übereinstimmung mit den Werten, die nach der Gleichung von Monrad und Pelton bei höheren Prandtl-Zahlen ermittelt wurden.

АНАЛОГИЯ МЕЖДУ ГИДРОДИНАМИЧЕСКИМ ТРЕНИЕМ И ТЕПЛОПЕРЕНОСОМ В КОЛЬЦЕВЫХ КАНАЛАХ

Аннотация—Из аналогии между гидродинамическим трением и теплопереносом получено уравнение для коэффициента теплопереноса на внутренней стенке турбулентного кольцевого канала. Сравнение данных, найденных по уравнению, с полученными автором экспериментальными результатами для воздушного потока показало хорошее совпадение. Уравнение хорошо согласуется также с уравнением, предложенным Монрадом и Пелтоном для более высокого числа Прандтля.